RAN-1901001101030001

RAN-1901001101030001

M. A. (Part - I) External - Examination March - 2026

Mathematics

Topology (Paper - 403)

Time: 3 Hours] **[Total Marks: 100**

સૂચના : / Instructions

(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the above details on your answer book
(2) Follow usual notations and conventions
(3) Figures to the right indicates marks
(4) Attempt All questions

Seat No.:

Student's Signature

Q. 1. (a) Define separable space. Prove that any co-finite topological space is separable. **07**
(b) Show that every second countable space is first countable. **07**
(c) If a metric space X has the Bolzano-Weierstrass property then prove that X is sequentially compact. **06**

OR

Q. 1. (a) Define topological space. Let T_1 and T_2 be topologies on a non-empty set X, prove that $T_1 \cap T_2$ is a topology on X. Is $T_1 \cup T_2$ a topology on X? Justify your answer. **07**
(b) Define closure of a set. Show that a set A is closed if and only if $A = \bar{A}$. Also, deduce that $\overline{A \cup B} = \overline{A} \cup \overline{B}$. Is $\overline{A \cap B} = \overline{A} \cap \overline{B}$? Justify.
(c) Prove that every separable metric space is second countable. **06**

Q. 2. (a) If a metric space X is complete and totally bounded, then prove that X is compact. **07**
(b) Let X_1 and X_2 be two topological spaces. Show that the product topology on $X = X_1 \times X_2$ is the weak topology generated by projections. **07**
(c) Define compact space. Show that a continuous real function f defined on a compact space X attains its infimum and its supremum. **06**

OR

Q. 2. (a) Prove that every sequentially compact metric space is totally bounded. **07**
(b) Let X be a second countable space. Prove that any open base for X has a countable sub class which is also an open base. **07**
(c) Define countably compact space. Prove that a second countable space is countably compact if and only if it is compact. **06**

Q. 3. (a) Prove that a topological space is a T_1 space if and only if each point is a closed set. **07**
(b) Prove that every compact metric space has the Bolzano-Weierstrass property. **07**
(c) Define Lebesgue number. Prove that every sequentially compact metric space is compact. **06**

OR

Q. 3. (a) Define normal space. Show that a closed subspace of a normal space is normal. **07**
(b) State and prove Tychonoff 's theorem. **07**
(c) Prove that a completely regular space is regular. **06**

Q. 4. (a) Prove that a topological space X is disconnected if and only if there exists a continuous mapping of X onto the discrete two-point space $\{0,1\}$. **07**
(b) Let X be a topological space. If $\{Ai\}$ is a non-empty class of connected subspace of X such that $\cap_i A_i$ is non-empty, then prove that $A = \cup_i A_i$ is also connected subspace of X. **07**
(c) Prove that the closure of connected set is connected. **06**

OR

Q. 4. (a) Prove or disprove that a connected subspace of a locally connected space is locally connected if X is real line. **07**
(b) Show that R^n and C^n are connected. **07**
(c) Define locally connected space. Let X be a locally connected space. If Y is an open subspace of X, show that each component of Y is open in X. In particular, each component of X is open. **06**

Q. 5. (a) Prove that every closed and bounded subspace of the real line is compact. **07**
(b) Prove that the topological space X is locally connected if the components of every open subspace of X are open in X. **07**
(c) Show that any continuous mapping of a compact metric space into a metric space is uniformly continuous. **06**

OR

Q. 5. (a) What is totally disconnected? Show that every discrete space is totally disconnected. **07**
(b) Define component of a topological space and show that each component of a topological space is closed. **07**
(c) Prove that the topological space X is locally connected if the components of every open subspace of X are open in X. **06**